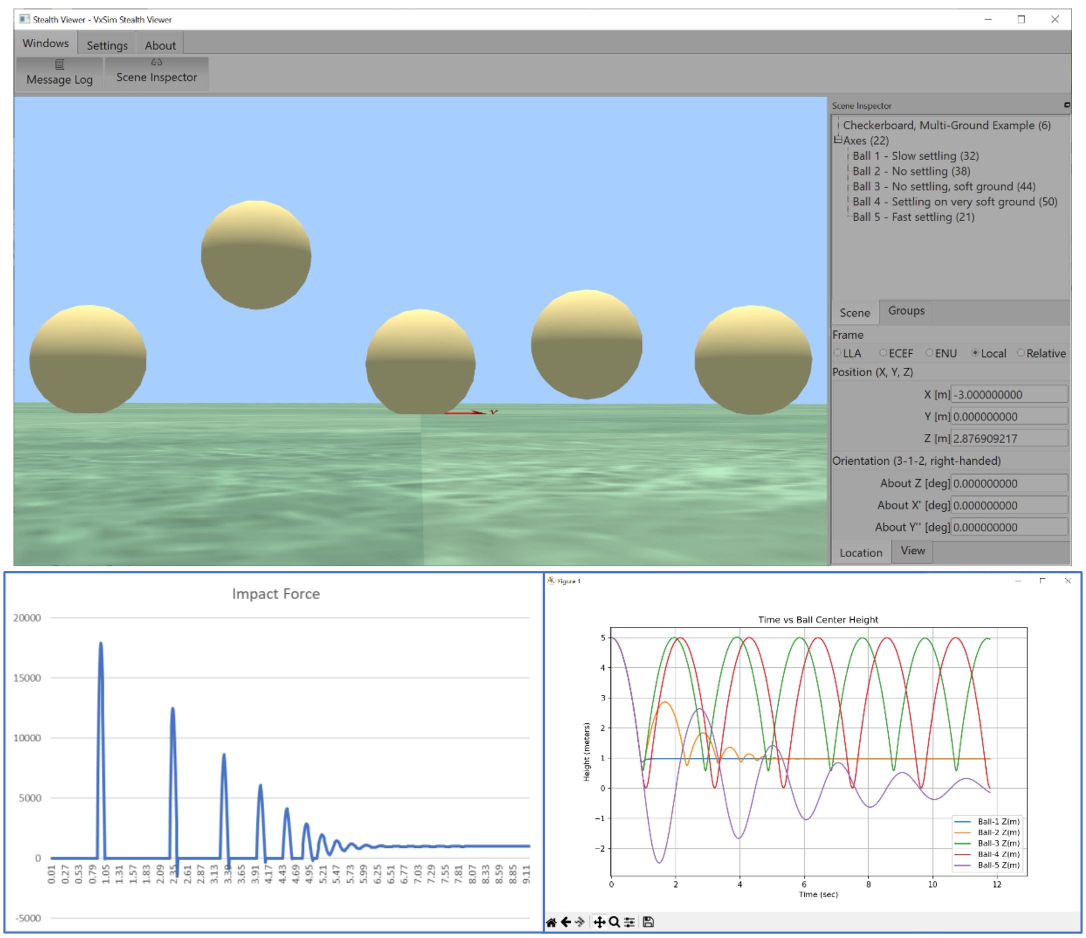
White Paper: Modeling Contact in VxSIM™

Accurately modeling the contact dynamics is crucial in fields such as physics simulation, robotics, and computer graphics. This paper presents an overview of different approaches to modeling contact, using a bouncing ball example, for fundamental exploration, comparing their advantages and limitations. We examine analytical, numerical, and empirical models, discussing their applications in various domains.

1. Introduction

Contact modeling for a bouncing ball involves determining the interaction forces between the ball and the surface upon impact. The complexity of this problem arises from factors such as material properties, deformation, energy dissipation, and numerical stability. In VxSIM™, these methods can be implemented using either the embedded SimCode process, included with all licenses, or by connecting third party applications such as MatLab, Python, Adams, Ansys, MotionGenesis, Chrono, etc., utilizing the Application Programming Interface (API) available in the Research and Professional licenses.



Bouncing Ball Example in VxSIM™, where SimCode is used to implement the equations of motion, solve with RK4 integrator and record the ball states (position, velocity, acceleration) and forces. Recording of data is user definable, such as comma separated variable (CSV) which is easily displayed via Excel or Python graphs.

2. Contact Modeling Approaches

2.1 Rigid Body Impact Models

These models assume the ball and surface are rigid, with no deformation. The impact is characterized by a coefficient of restitution (COR), which defines the ratio of post-impact to pre-impact velocity.

- **Newton's Law of Restitution:** Defines the velocity change based on a restitution coefficient.
- **Poisson's Hypothesis:** Uses a two-stage compression-restitution process to model impact.
- **Limitations:** Fails to capture detailed contact force evolution and deformations.

2.2 Compliance-Based Models

These models consider elasticity and deformation during contact.

- **Hertzian Contact Theory:** Models force as a function of deformation using Hertz's nonlinear elasticity theory and relies on **Young's modulus** and **Poisson's ratio**.
- **Kelvin-Voigt Model:** Uses a spring-damper system to capture elastic and damping effects.
- **Elastic and Plastic Deformation:** For more advanced modeling, plastic deformation is considered in **Finite Element Analysis (FEA)**, which accounts for yield strength and permanent deformation.
- **Limitations:** Can be computationally expensive and require precise material parameters.

2.3 Impulse-Based Models

These models compute impulses instead of forces, updating velocities instantaneously.

- **Discrete Impulse Models:** Apply instantaneous velocity updates based on collision detection.
- **Penalty-Based Impulse Models:** Introduce small forces over a finite time to approximate impulse effects.
- **Limitations:** May introduce artificial numerical oscillations and stability issues.

2.4 Finite Element and Continuum Models

These methods use detailed numerical simulations of deformation and stress distribution.

- **Finite Element Analysis (FEA):** Simulates detailed material deformations during impact and accounts for both **elastic and plastic** behaviors.
- **Smoothed Particle Hydrodynamics (SPH):** Uses particle-based methods for soft-body interactions.
- **Limitations:** Computationally intensive, requiring high-resolution meshes or particles.

2.5 Data-Driven and Machine Learning Models

Recent advancements in machine learning allow empirical modeling based on experimental data.

- **Neural Networks:** Train models on real-world impact data to predict post-collision dynamics.

- **Reinforcement Learning:** Learns optimal impact response policies in dynamic environments.
- **Limitations:** Requires extensive data and lacks physical interpretability.

3. Numerical Methods and Stability Considerations

Numerical integration methods are crucial in simulating bouncing ball dynamics, particularly when solving differential equations governing motion and deformation.

- **Euler Method:** A first-order numerical approach, simple but prone to numerical instability for stiff systems.
- **Runge-Kutta Method (RK4):** A higher-order method that provides better accuracy and stability for contact dynamics.
- **Runge-Kutta 8 (RK8):** An even higher-order method offering extremely high precision. However, it requires significantly more function evaluations per step than RK4, making it computationally expensive. While RK8 is useful for applications demanding very fine error control, such as astrophysical simulations, it is often excessive for bouncing ball simulations where computational efficiency is a concern.
- **Adams-Bashforth Method:** A multi-step explicit method that improves efficiency by utilizing previous time steps to predict future states. It requires fewer function evaluations per step than Runge-Kutta methods but may struggle with rapid impact dynamics. Since it is an explicit method, it is not well-suited for stiff problems, where implicit methods like **Adams-Moulton** or **Backward Differentiation Formulas (BDF)** are preferred.
- **Natural Frequency and Time Step Size:** The time step should be small relative to the system's **natural frequency** to ensure numerical stability. If the time step is too large, high-frequency oscillations and energy errors can arise, particularly in contact and deformation models.
- **Stiff Systems:** A system is considered **stiff** when it contains rapidly changing components that require very small time steps for numerical stability. These systems often involve high-frequency dynamics, such as elastic deformations in contact modeling, where explicit numerical methods (e.g., Euler) struggle due to stability constraints, necessitating the use of implicit methods or adaptive time stepping.
- **Implicit Methods:** Implicit numerical schemes, such as **Backward Euler** and **Implicit Runge-Kutta (e.g., Radau IIA)**, improve stability by solving equations that account for future states. These methods allow for larger time steps when dealing with stiff systems.
- **Adaptive Time Stepping:** Methods like **Dormand-Prince RK45** and **Gear's BDF (Backward Differentiation Formula)** adjust the time step dynamically based on error estimates. This approach ensures efficiency by taking larger steps in smooth regions and smaller steps during rapid transitions, such as impact events.

4. Comparative Analysis

Each method has trade-offs between accuracy, computational efficiency, and applicability. For real-time simulations, rigid body models with COR tend to be preferred, whereas high-fidelity applications in engineering demand compliance-based or FEA models. Numerical methods must be carefully selected to balance accuracy and stability, ensuring that the chosen time step size is appropriate for the system's natural frequency.

4.1 Connection to Vehicle Tire-Terrain Interactions

Many of the methods discussed for modeling bouncing ball contact can be extended to vehicle tire-terrain interactions, particularly in off-road environments. Compliance-based models, such as Hertzian contact theory, are often used to describe the deformation of tires in contact with soft ground. Additionally, empirical and physics-based models such as **Bekker's theory** and **Wong's terramechanics model** describe soil-tire interaction by considering parameters like soil cohesion, shear strength, and sinkage.

- **Bekker's Model:** Utilizes pressure-sinkage relationships to determine terrain deformation under tire load, similar to how compliance-based models predict ball deformation.
- **Wong-Reece Model:** Extends Bekker's approach by incorporating shear stress and slip characteristics, providing a more comprehensive framework for dynamic tire-soil interaction.
- **FEA for Tire Dynamics:** Just as FEA is used for detailed ball impact analysis, it is also applied in high-fidelity tire modeling to capture elastic and plastic deformation in different terrain conditions.
- **Numerical Considerations:** Like bouncing ball simulations, tire-terrain models must balance accuracy and computational efficiency. Stiff numerical systems emerge when modeling rapid load changes, necessitating implicit solvers and adaptive time-stepping methods to ensure stability.

5. Conclusion

Selecting an appropriate contact model is a function of both the desired accuracy and computational efficiency required for a given application. Rigid body models with simple restitution coefficients are well-suited for real-time simulations, while compliance-based, finite element, and data-driven models provide higher fidelity at increased computational costs.

VxSIM™ provides a robust simulation framework with a modular architecture that supports a wide range of dynamic systems, including land, water, and air vehicles, as well as articulated multibody structures such as robotic arms and advanced suspension systems. Current work focuses on integrating full stack control systems, including autonomy stacks, next generation path planners, communications, and human systems interfaces, into these simulations (i.e., software/hardware-in-the-loop) to enhance predictive capabilities while conducting behavioral analysis in complex mission space with many collaborative systems.

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